JHEP HYPER VERSION

Received: *April 3, 2000,* Accepted: *April 6, 2000*

Renormalization group flows from gravity in anti-de Sitter space versus black hole no-hair theorems

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Abstract: Black hole no-hair theorems are proven using inequalities that govern the radial dependence of spherically symmetric configurations of matter fields. In this paper, we analyze the analogous inequalities for geometries dual to renormalization group flows via the AdS/CFT correspondence. These inequalities give much useful information about the qualitative properties of such flows. For Poincaré invariant flows, we show that generic flows of relevant or irrelevant operators lead to singular geometries. For the case of irrelevant operators, this leads to an apparent conflict with Polchinski's decoupling theorem, and we offer two possible resolutions to this problem.

Keywords: String Duality, Brane Dynamics in Gauge Theories.

JHEP04(2000)011

Contents

1. Introduction

The conjectured correspondence between type-IIB string theory on a $AdS_5 \times S^5$ background, and large N SU(N) maximally supersymmetric Yang-Mills theory, promises to offer new insights into conventional quantum field theory (see [1] for a general review). In the large N limit, with strong 't Hooft coupling $g_{YM}^2 N$, the correspondence provides us with a description of the super Yang-Mills theory in terms of collective coordinates, which are simply the fields of compactified type-IIB supergravity. This leads to a new kind of strong coupling expansion, which has the advantage that four-dimensional Poincaré invariance is manifest.

Gauge theories with fewer or no supersymmetry are of much greater physical interest. These may be studied by perturbing the couplings of the maximally supersymmetric Yang-Mills theory at an ultraviolet scale, and studying the renormalization group flow to the infrared. By the AdS/CFT correspondence these RG flows are dual to asymptotically anti-de Sitter geometries. The UV perturbation appears as boundary conditions on the supergravity fields at large radius, and the radial dependence of the flow as one moves into the interior of the space encodes the RG flow to the IR.

It has been argued that flows involving just the relevant and marginal couplings of the Yang-Mills theory may be studied by truncating to five- dimensional gauged supergravity (see [1] for a discussion and further references). Numerous gravity duals of RG flows have been constructed within this framework [2]–[7].

The geometries involved have a close relation to the large mass limit of black hole geometries. In fact, many of the examples in the previous paragraph can be thought of precisely in this way. It is natural then to ask what the analogs of the black hole no-hair theorems have to say about the geometries dual to Poincaré invariant RG flows. This will be the main focus of this paper.

Black hole no-hair theorems apply to asymptotically flat spherically symmetric black holes in theories of gravity coupled to scalar fields. Some helpful reviews of this subject are [8]. Also see [9, 10] for modern versions of the theorems. The theorems place conditions on the interaction potential of the scalar fields: that it be positive semidefinite [9], or convex [10]. The theorems then proceed by deriving inequalities which govern the radial dependence of the scalars, with the result that scalar hair cannot depend on any continuous parameters at infinity. These results have been generalized to spherically symmetric black holes in anti-de Sitter space in [11].

The scalar potential of compactified supergravity does not satisfy the constraints used in the above theorems. The scalar potential appearing in the five-dimensional gauged supergravity, for example, is not positive definite, and has directions where the potential asymptotes to either $+\infty$ or $-\infty$. When we follow through the same conditions in the context of the RG flow geometries, we will find hair is allowed, at least for finely tuned choices of the perturbations. Nevertheless, the analogs of the inequalities on the flow of the scalars will provide us with much useful qualitative information about the properties of these flows, generalizing the c-theorem of [4, 5].

For generic perturbations in the UV, we find the gravity dual of the RG flow becomes singular in the interior. For relevant operators this is perhaps not such a big surprise — from the field theory point of view, a generic perturbation will lead to a theory that is either free or trivial (confining) in the infrared. Since gravity is inherently a non-linear theory, the only way it can accommodate this is for the geometry to become singular. For irrelevant operators, on the other hand, this is more troubling, as we find that the singularity will depend in details on the UV perturbation of the irrelevant couplings, of which we have an infinite number as $N \to \infty$. At first sight, this looks like a violation of Polchinski's theorem [12], which states that all couplings in the infrared should be determined by the finite number of relevant and marginal couplings alone.

We offer two possible explanations of this fact: either a large class of irrelevant operators in the super Yang-Mills are dangerous in the sense that they become relevant along a typical RG flow; or the dependence of the supergravity solution on the scalar dual to an irrelevant operator is a red herring as far as the infrared physics goes, and this dependence goes away when the stringy resolution of the singularity is properly understood. The first possibility seems to indicate the large N Yang-Mills theory is sick in the sense we have a massive failure of the decoupling of irrelevant operators from low energy physics. The second possibility is more desirable from the quantum field theory point of view, but indicates some spectacular stringy miracles are needed to properly understand the infrared physics.

2. Inequalities for RG flows in AdS/CFT

For gravity duals of Poincaré invariant RG flows, the five-dimensional asymptotically anti-de Sitter part of the spacetime will have a metric of the form

$$ds^2 = e^{2A(r)}(dx^\mu)^2 - dr^2\,, \tag{2.1}$$

where $\mu = 0, 1, 2, 3$. We will be interested in solutions where this metric couples to scalar fields, with an action of the form

$$S_\phi = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{|g|} \Big(-R + \frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - V(\phi) \Big) . \tag{2.2}$$

The generalization to multiple scalar fields is immediate, assuming the kinetic term is positive definite. We will adopt conventions where $\kappa_5 = 1$, and choose the negative cosmological constant (the constant term in V) so that $A = r$ is the vacuum solution as $r \to \infty$.

We wish to view the renormalization group flow in the Wilsonian sense, so rather than trying to introduce counter-terms and sending the UV cutoff to infinity, we will work with an explicit UV cutoff. Thus we will impose boundary conditions on the scalar fields at some fixed, large value of $r = r_{\mathrm{UV}}$. Near this value of r we will require that the scalar field perturbation is small, so that the vacuum Einstein equations hold (i.e. $A \sim r$).

There are two independent boundary conditions possible for scalar fields:

$$\phi = \phi_+ \exp(\alpha_+ r) + \phi_- \exp(\alpha_- r) . \tag{2.3}$$

where $\alpha_\pm = -2 \pm 2\sqrt{1 + (m^2/4)}$, for a scalar of mass m. For relevant operators $m^2 < 0$, but satisfies a unitarity bound [13] provided $m^2 > -4$. This bound guarantees that the energy of fluctuations of the field is positive definite. Marginal operators have $m^2 = 0$, while irrelevant operators have positive m^2. Examples of irrelevant operators with finite conformal dimensions are chiral primaries dual to massive Kaluza-Klein modes of type-IIB supergravity on $AdS_5 \times S^5$. Stringy modes will typically have conformal dimensions that diverge in the large N limit, [14].

In order to deform the lagrangean of the Yang-Mills theory in the UV by a coupling dual to one of these modes, the ϕ_+ part of the solution should be the dominant contribution at $r = r_{\mathrm{UV}}$. This is to ensure the correct two-point correlation function is reproduced, as explained in details in [15]. There they also point out for conformal dimensions Δ between 2 and 3, either possibility is allowed, corresponding to inequivalent quantizations. This subtlety will not be a concern for us, as we will be mostly interested in deformations by irrelevant operators $\Delta > 4$. Turning on the ϕ_- boundary condition corresponds to considering a non-trivial vacuum state in the strong coupling large N Yang-Mills theory, which gives rise to expectation values for operators dual to the gravity mode.[1] The gravity solutions found in [16, 17, 18, 19] correspond to flows of this type.

[1]The mapping of ϕ_+ and ϕ_- to perturbations and expectation values, respectively, in the field theory dual is precise only when the cutoff $r_{UV} \to \infty$. For finite r_{UV} it is not known in general how to make a precise distinction between the two. We comment further on this point in the following section.

In quantum field theory, perturbations by irrelevant couplings drastically alter the UV behavior of the theory. For us, this shows up as the large r asymptotics of the spacetime being altered, if we extrapolate past $r_{\rm UV}$. However by imposing our boundary conditions at finite r there will be a scaling regime where for small perturbations the geometry will be approximately AdS near the boundary $r_{\rm UV}$. It will not be possible to remove the UV cutoff, keeping the coefficient of the irrelevant coupling fixed. Of course this is precisely the situation in the usual wilsonian renormalization group.

Following Sudarsky's proof of the black hole no-hair theorems [9] we will now derive an equality the scalar solution must satisfy for any RG flow. Take $\phi = \phi(r)$, and consider the equation of motion of the scalar

$$e^{-4A}(e^{4A}\phi')' = \frac{\partial V}{\partial \phi}\,, \tag{2.4}$$

where $'$ denotes a derivative with respect to r. Multiply this by ϕ', and define the "energy"

$$E = \frac{1}{2}(\phi')^2 - V(\phi) - 6\,, \tag{2.5}$$

to obtain

$$E' = -4A'(\phi')^2\,. \tag{2.6}$$

The -6 in (2.5) simply shifts our definition by an irrelevant constant for convenience.

Now the c-theorem of [4] implies $A'' \leq 0$ for all r, provided ϕ satisfies a version of the weak energy condition. Thus if $A' > 0$ at $r = r_{\rm UV}$, $A' > 0$ for all $r < r_{\rm UV}$. Therefore our general inequality for scalar perturbations is

$$E' \leq 0 \tag{2.7}$$

for all $r < r_{\rm UV}$.

Next we examine the boundary conditions on the fields to check whether non-trivial flows can exist which are compatible with (2.7). For the tachyonic scalars (2.7) is easy to satisfy. In that case we can send $r_{\rm UV}$ to infinity in a smooth way, so we know $E_{\rm UV} = 0$. The condition (2.7) then implies that in the infrared $E_{\rm IR} > 0$. Even if $\phi' \to 0$ in the infrared, this condition can still be satisfied because V is not bounded below. Thus these solutions can display non-trivial hair. The solutions found in [2]–[7] are of this type.

For massive scalars the potential is bounded below (ignoring for the moment higher non-linear couplings to lighter fields, which should be valid for sufficiently small perturbations). However now we are not able to take our boundary $r_{\rm UV}$ off to infinity. Instead we find $E_{\rm UV} = -2\alpha_+\,\phi_+^2 \exp(2\alpha_+ r_{\rm UV}) < 0$. Condition (2.7), implies $E_{\rm IR} > E_{\rm UV}$ which again is possible to satisfy for some solutions, so once again non-trivial hair is possible.

JHEP04(2000)011

On the other hand, if we impose that the massive scalar field approach the ϕ_- solution in the UV, we are free to scale $r_{\rm UV}$ to infinity. Now $E_{\rm UV}=0$, and at least if ϕ decouples from the other scalars in the potential, and starts out at a global minimum at infinity, the only possible solution is to have ϕ be constant. Thus no hair is possible, which implies that states in the Yang-Mills theory with expectation values turned on for such irrelevant operators will not have good gravity duals. Of course such Poincaré invariant vacuum states are certainly not present in finite N Yang-Mills theory, so this is a nice consistency check.

To study the non-trivial solutions further, we will need some linear combinations of the Einstein equations

$$\begin{aligned} G^r_r = \kappa_5^2 T^r_r &\Longrightarrow -6(A')^2 = -\frac{1}{2}(\phi')^2 + V(\phi)\,, \\ G^t_t - G^r_r = \kappa_5^2(T^t_t - T^r_r) &\Longrightarrow -3A'' = (\phi')^2\,. \end{aligned} \tag{2.8}$$

The c-theorem of [4] follows immediately from the last of these equations. Solving the first of these for A' and taking the positive root, as required by the c-theorem, and inserting into (2.4) yields

$$\phi'' + \frac{4}{\sqrt{6}}\phi'\sqrt{\frac{1}{2}(\phi')^2 - V(\phi)} = \frac{\partial V}{\partial \phi}\,. \tag{2.9}$$

The term in the square root is just $E(r)+6$, so by (2.7) this is always positive along the flow, provided our UV perturbations are small. Equation (2.9) now has a nice physical interpretation. It is simply the equation governing a particle moving in the potential $-V$ with "negative" position dependent damping as we flow from the UV to the IR. We can therefore immediately conclude that for generic boundary conditions the negative damping will cause the solution to become singular in the infrared. Fine tuning of the initial perturbation in the UV will be needed to land the solution on an extremum of the potential, to obtain a supergravity solution that smoothly interpolates from the UV to the IR.

For the case of relevant operators examples of such singular flows have been constructed in [2]–[7]. The singularities of such flows exhibit a certain universality as emphasized in [20]. Namely, for a wide range of parameter choices, the potential becomes irrelevant near the singularity, which appears at some finite radius $r=r_0$. Solving (2.9) and (2.8) yields

$$\phi \sim \pm\sqrt{\frac{3}{4}}\log(r-r_0)\,, \qquad A \sim \frac{1}{4}\log(r-r_0)\,, \tag{2.10}$$

near the singularity. This singularity leads to a curvature singularity both in the string and the Einstein frame metrics, from the five- and ten-dimensional points of view. For multiple scalar fields, one obtains the same behavior, with $\phi^a \sim n^a \log(r-r_0)$ for some suitably normalized vector n^a, whose direction will depend in details on the initial perturbation in the UV.

The same statements will apply also for the case of flows from perturbations by a large class of irrelevant operators, because only the scalar kinetic term is relevant in this limit. In general, it is easy to find regions in the parameter space of initial conditions where the n^a will depend in details on the irrelevant couplings in the UV.

In certain regimes of the parameter space of the UV perturbations, a different kind of singularity may arise where the potential of the scalar fields dominate the flow. This is always the case for flows which preserve supersymmetry. Such flows will depend on the details of the potential in the highly non-linear regime, so we will not comment on them further here.

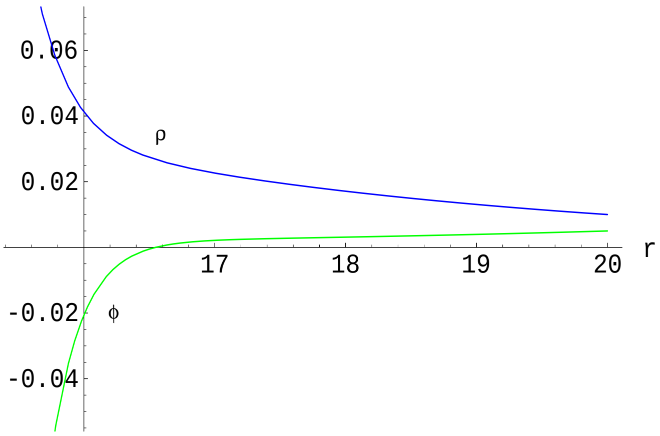

Figure 1: The scalar $\phi(r)$ dual to an irrelevant coupling, together with a scalar $\rho(r)$ dual to a relevant coupling.

Let us now consider a concrete example of a flow of an irrelevant operator dual to a massive Kaluza-Klein mode ϕ coupled to a relevant operator, dual to a tachyonic mode ρ. Unfortunately the non-linear couplings of the massive Kaluza-Klein modes in the $AdS_5 \times S^5$ compactification of type-IIB supergravity are not known. See [21] for recent developments in this direction. As a toy model for the flow, take the potential to be $V = (1/2)\phi^2 - (1/2)\rho^2 - 6$, and ignore the non-linear couplings not already present in (2.9). Boundary conditions are chosen at $r = r_{\mathrm{UV}}$ so that $\phi_- = \rho_- = 0$ and $\phi_+,\ \rho_+ \ll 1$. The results of the numerical evolution toward the infrared are shown in figures 1 and 2. The singular behavior is precisely of the form (2.10), (on the $+$ branch).

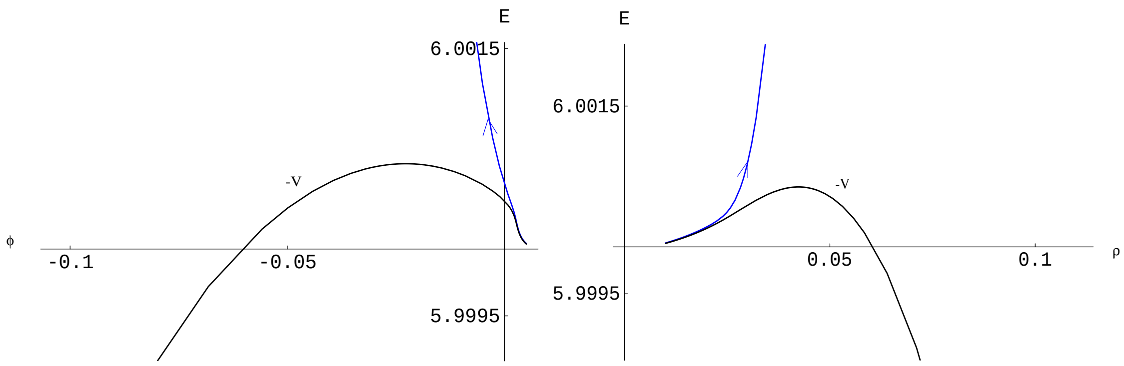

Figure 2: On the left, the trajectory in the E, ϕ plane, together with the potential $-V$. On the right, the corresponding trajectory in the E, ρ plane.

By making the perturbations sufficiently small, we can obtain an arbitrarily large scaling region, where the relevant coupling is increasing toward the infrared, while the irrelevant coupling is decreasing. However when the relevant coupling dual to ρ becomes sufficiently large, it pushes both ρ and ϕ into the runaway behavior (2.10). For reasonable choices of parameters, ϕ is of the same order as ρ near the singularity.

If m^2 of the mode ϕ is taken to be large (e.g. we considered the same example for $m^2 = 100$), the irrelevant mode decreases more rapidly along the flow. However, because the ρ field is monotonically increasing as r decreases, eventually the coupling to ϕ induces a mixing of the IR stable solution of the linearized equation with the IR unstable solution. For large m^2 the unstable mode increases correspondingly rapidly. Numerical evidence points to the fact that the n^a characterizing the runaway behavior (2.10), are typically dominated by the participating fields with the largest value of m^2.

3. Discussion

For perturbations by relevant couplings we expect a large change in the supergravity solution as we flow to the IR. However if the solution becomes singular for generic perturbations it is still a bit puzzling. Of course from the field theory point of view it is natural that we should need to fine tune couplings in the UV to land on a non-trivial conformal field theory in the IR. A generic relevant perturbation, on the other hand, will cause the theory to flow to either a free fixed point in the IR, or a trivial theory if all the physical degrees of freedom become massive. At least for free fixed points, gravity will have a hard time reproducing the behavior as it is inherently non linear. The only way for the supergravity solution to circumvent this fact is for the solution to become singular. The singularity associated with free fixed points may be resolved by string theory in the tensionless limit as suggested in [22], for the case of free $N = 4$ Yang-Mills.

For perturbations by irrelevant couplings the generically singular behavior is rather more surprising from the point of view of renormalization group flow.[2] One expects a small perturbation by an irrelevant operator in the UV to produce a small change in the IR (assuming that operator does not flow to become relevant). The precise statement is summarized by Polchinski's theorem [12], which says that along a renormalization group flow down to some scale Λ, the unrenormalizable couplings at the scale Λ are determined in terms of the renormalizable couplings at Λ. The flow is to a stable surface specified by the finite number of renormalizable couplings. On the other hand, from the gravity point of view, the flow to the IR depends on the value of a potentially infinite number of non-renormalizable couplings for $N \to \infty$, apparently contradicting Polchinski's theorem.

[2] An example of a full ten-dimensional supergravity dual of such a flow can be found in [23].

One possible resolution of this problem is as follows: the large N Yang-Mills theory at strong coupling is plagued with an abundance of dangerous irrelevant operators, which become relevant along a generic renormalization group flow. While this behavior is consistent with Polchinski's theorem, it implies we will have a hard time decoupling high dimension operators from low energy physics. This rather limits the utility of truncations to five- dimensional gauged supergravity, which describes only the relevant and marginal couplings in the UV.

Another possible resolution is the following: to understand the correct IR physics we need to be in a region where curvatures become of order the string scale. String theory resolves these curvature singularities by correctly matching onto the highly non-universal supergravity solution, but all the universal features of the flow to the IR are captured only in the region where stringy effects are large. This seems like a tall order, as it requires that all the low energy observables be independent of the details of the supergravity solution in the region of sub-stringy curvatures in the vicinity of the singularity. However there are examples where behavior of this type is at least partially realized. If this possibility is realized, it seems to imply we learn little about the IR physics of large N super Yang-Mills by studying supergravity solutions alone.

One such example is extracting glueball masses from these singular geometries. In [18] this calculation is performed for glueballs dual to axion modes. The axion, and a large class of other modes, see a repulsive potential as the singularity is approached. The mass spectrum of glueballs dual to these fluctuations is independent of the details of the singularity. However, in the example considered in [18] it is not hard to find other modes which see no repulsive potential. The dilaton is one example. Without a resolution of the singularity it is not possible to compute the spectrum of such fluctuations.

It is difficult to make general statements about how string theory may resolve these singularities. One class of singularities that have been studied in details are the repulson singularities that arise in D-brane configurations dual to large N SU(N) $\mathcal{N} = 2$ supersymmetric Yang-Mills theory [24]. Here a naked singularity is resolved by the formation of a smooth distribution of D-brane charge. The basic idea here is likely to generalize to a large class of singularities — brane charge spreads out, giving a smooth configuration (see also [25] for studies of this effect). From the supergravity point of view explicit sources are required in the supergravity equations of motion to represent the presence of the D-branes. If stringy resolutions of this type are relevant for the singularities we have discussed here, it supports the viewpoint that stringy effects mask the IR physics from the dependence on the irrelevant coupling parameters.

One should also bear in mind the additional parameters in the supergravity solutions corresponding to expectation values of operators. As mentioned above, the mapping between boundary conditions at $r = r_{UV}$ and couplings and expectation values in the field theory becomes difficult for the singular geometries under consideration here. As far as we know, the finite N CFT does not possess a moduli space of vacua corresponding to expectation values of non-marginal operators. The supergravity dual of such a configuration, on the other hand, appears to be well-defined away from the vicinity of the singularity. This suggests that once the string theory resolution of these singularities is understood, a large class of these singularities will be found to be unstable. Presumably the stability of the singularity will provide the extra information needed to uniquely determine the boundary conditions at $r = r_{UV}$ corresponding to pure coupling perturbations. We emphasize this problem cannot be analyzed within the framework of low-energy supergravity alone, as in general boundary conditions must be imposed at the singularity to determine the future evolution. A less attractive possibility is that these extra "moduli" are an artifact of the large N limit.

One may wonder what the cosmic censorship conjecture has to say about these singular solutions. See [23] for a review of this topic. The weak cosmic censorship conjecture roughly states that for generic smooth initial data, in asymptotically flat space, with suitable matter, any singularities that form are necessarily hidden behind event horizons. There is much nontrivial evidence for this conjecture, and no clear counter-examples. However, there is strong evidence that for spherically symmetric collapse of scalar fields in asymptotically flat spacetime this weak cosmic censorship conjecture holds [24]. One difference in our case is the negative cosmological constant. The stress-energy tensor (including the cosmological constant term) violates the dominant energy condition by allowing negative energy densities. This inhibits the formation of black hole horizons, permitting naked singularities to appear in the evolution of scalar fields. For us, perhaps the most important difference is that four-dimensional Poincare invariance of the solutions violates the condition that the initial data be generic.

One might worry that the boundary conditions for irrelevant couplings correspond to infinite energies at infinity. However we insist on placing these boundary conditions at finite radius. These solutions might then be viewed as the interiors of smooth solutions which asymptote to anti-de Sitter space. Specifying the exterior solution corresponds in field theory language to specifying counter-terms in the bare action needed to keep physical quantities finite as the cutoff is removed.

It is interesting to consider the finite temperature version of inequality (2.7), and to test whether the singular solutions we have described are stable with respect to turning on this additional parameter. We can repeat the argument of the previous section for the metric

$$ds^2 = e^{2A(r)}(\mu(r)dt^2 - (dx^i)^2) - \frac{dr^2}{\mu(r)}, \tag{3.1}$$

where $i = 1, 2, 3$ and $\mu \sim 1 - M \exp(-4r)$ at large r. The scalar field equation motion now becomes

$$(\mu\phi')' + 4A'\mu\phi' = \frac{\partial V}{\partial \phi}\,. \tag{3.2}$$

Multiplying by ϕ' and rearranging gives

$$\frac{1}{2}\left(\mu(\phi')^2\right)' + \left(\frac{1}{2}\mu' + 4A'\mu\right)(\phi')^2 = \frac{\partial V}{\partial \phi}\phi'\,. \tag{3.3}$$

We can now define an energy function

$$E = \frac{1}{2}\mu(\phi')^2 - V - 6\,, \tag{3.4}$$

which then satisfies, using (3.3)

$$E' = -\left(\frac{1}{2}\mu' + 4A'\mu\right)(\phi')^2\,. \tag{3.5}$$

Using Einstein equations, it follows that each term on the right hand-side of (3.5) is negative outside the horizon. Thus we obtain the same condition for the new E (2.7).

At a regular black hole horizon, ϕ' will be finite, while μ will vanish. Thus $E = -V(\phi_{\rm hor}) - 6$ at the horizon. The conditions on E near the boundary are the same as before, namely for a relevant perturbation we can assume $E_{\rm UV} = 0$, or for a irrelevant perturbation $E_{\rm UV}$ is small and negative. A smooth black hole will exist provided $V(\phi_{\rm hor}) + 6 < -E_{\rm UV}$, which is easy to satisfy for the tachyonic relevant modes, and possible to satisfy for irrelevant modes.

However we do not see any sign that the singular solutions we have considered at zero temperature ($M = 0$) will, in general, become hidden behind event horizons when we turn on M. To answer this question in more details, we have numerically solved Einstein equations coupled to a scalar generalizing the example of figure 1 to finite M, and found that the singularity persists until M becomes larger than some finite value. The singularities are therefore stable against turning on an additional parameter, which points toward the fact that they are indeed physically meaningful.

While this manuscript was being prepared [28] appeared, where very similar methods are used with a rather different emphasis.

Acknowledgments

I wish to thank A. Jevicki, R. Myers, S. Ramgoolam and especially Radu Tatar for helpful discussions. This research is supported in part by DOE grant DE-FE0291ER40688-Task A.

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